Section 2.1: Properties of Matrix Operations

Theorem 2.4: Throughout this result, the symbol $\mathbf{0}$ represents the $m \times n$ zero matrix. The symbols 0 and 1 are used to denote the real numbers zero and one.

1. For all $A \in M_{m,n}(\mathbb{R})$, $0 \cdot A = \mathbf{0}$.
2. For all $A \in M_{m,n}(\mathbb{R})$, $1 \cdot A = A$.
3. For all $\alpha \in \mathbb{R}$, $\alpha \cdot \mathbf{0} = \mathbf{0}$.
4. For all $A, B \in M_{m,n}(\mathbb{R})$, $A + B = B + A$ (commutativity)
5. For all $A, B, C \in M_{m,n}(\mathbb{R})$, $(A + B) + C = A + (B + C)$ (associativity)
6. For all $A \in M_{m,n}(\mathbb{R})$ and all $\alpha, \beta \in \mathbb{R}$, $(\alpha + \beta)A = \alpha A + \beta A$ (distributivity)
7. For all $A, B \in M_{m,n}(\mathbb{R})$ and all $\alpha \in \mathbb{R}$, $\alpha(A + B) = \alpha A + \alpha B$ (distributivity)
8. For all $A \in M_{m,n}(\mathbb{R})$ and all $\alpha, \beta \in \mathbb{R}$, $(\alpha\beta)A = \alpha(\beta A)$.
9. For all $A \in M_{m,n}(\mathbb{R})$, $A + (-A) = (-A) + A = \mathbf{0}$.

Properties of Matrix Multiplication

Theorem 2.10:

1. For all $A \in M_{m,n}(\mathbb{R})$ and all $k \geq 1$, $A\,\mathbf{0}_{n\times k} = \mathbf{0}_{m\times k}$ and $\mathbf{0}_{k\times m}\,A = \mathbf{0}_{k\times n}$.
2. For all $A \in M_{m,n}(\mathbb{R})$, $I_m\,A = A$ and $A\,I_n = A$.
3. In particular, if A is an $n \times n$ matrix, $I_n\,A = A\,I_n = A$.
4. If $A \in M_{m,n}(\mathbb{R})$, $B \in M_{n,k}(\mathbb{R})$, and $C \in M_{k,l}(\mathbb{R})$, then $A(BC) = (AB)C$. (associativity)
5. If $A \in M_{m,n}(\mathbb{R})$ and $B, C \in M_{n,k}(\mathbb{R})$, then $A(B + C) = AB + AC$. (distributivity)
6. If $A \in M_{m,n}(\mathbb{R})$, $B \in M_{n,k}(\mathbb{R})$, and $\alpha \in \mathbb{R}$, then $A(\alpha B) = \alpha(AB) = (\alpha A)B$.